# Polarization degeneracy at Bragg reflectance in magnetized photonic crystals

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A study is presented of the band structure in one-dimensional photonic crystals with anisotropic and gyrotropic layers. It is predicted that magnetization of the structure in a direction normal to the plane of the layers causes the formation of an additional band gap of a new type. The phenomenon is caused by the Bragg resonance between harmonics of different polarization and stems from the hybridization of these harmonics. Though the application of a magnetic field generally results in degeneracy lifting, in anisotropic magnetophotonic crystals a magnetic field may actually induce polarization-degenerate Bragg reflections. Moreover, while in a nonmagnetized photonic crystal, the Bloch waves of different polarization may have noncoincident band edges, the band gaps predicted herewith are shared by the Bloch waves of any polarization. This allows the design of polarization-independent optical tuning devices. Thus the formation of these band gaps enables the magnetic control of arbitrarily polarized light.

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## I. INTRODUCTION

Recent technological developments have enabled the fabrication of new types of artificial media. In this regard inhomogeneous structures deserve particular attention as their specific properties can extend beyond those of natural materials. Photonic crystals constitute one such structure. They are characterized by a periodic distribution of local constitutive parameters. This periodicity causes the resonant Bragg reflection of electromagnetic waves and the formation of optical band gaps.[1] The formation of optical band gaps is one of the features that make photonic crystals widely useful. The manipulation of these band gaps is of great technological importance for optical waveguiding, photon trapping, and optical filtering.

One of the ways to realize such manipulation at optical frequencies is to utilize a magneto-optical material as photonic crystal component.[2–9] The embedding of magneto-optic inclusions into the photonic crystal allows the control of its spectral properties: one may shift the dispersion curve and transmittance spectrum by application of a magnetic bias. As a consequence, for certain polarizations some frequencies lying near the band edge move away from a passband condition and into the band gap. However, the mere embedding of a magneto-optical material does not guarantee control of arbitrarily polarized light. In particular, the dispersion curves and, as a consequence, the band gaps of oppositely polarized waves (right and left helicities), for example, detune in opposite directions on the application of a magnetic field.

In the present communication we consider the case when the application of a magnetic bias field not only shifts the band gaps but also engenders the formation of a band gap of a special type. This phenomenon happens for magnetization normal to the layers in an anisotropic magnetophotonic crystal. This type of band gap appears synchronously for all Bloch wave polarizations, and so it may be useful for simultaneously controlling waves of different polarizations.

## II. FORMATION OF STANDARD BAND GAPS AT BRILLOUIN-ZONE BOUNDARIES

Below, without loss of generality, we consider waves traveling normally to the layers. For the convenience of the reader and for comparison with subsequent results presented below, we reproduce in this section certain results presented in Refs. 10 and 11 about band-gap formation.

According to the Floquet-Bloch theorem the eigensolution to Maxwell's equations in a system with periodic distribution of constitutive parameters is a Bloch wave,

$$\begin{aligned}\vec{E}(\vec{r}) &= \vec{f}(\vec{r})e^{i\vec{k}_{\mathrm{Bl}}\vec{r}} \\ &= \left(\sum_n \vec{f}_n e^{i\vec{G}_n\vec{r}}\right)e^{i\vec{k}_{\mathrm{Bl}}\vec{r}} \\ &= \sum_n \vec{f}_n e^{i(\vec{G}_n+\vec{k}_{\mathrm{Bl}})\vec{r}} \\ &= \sum_n \vec{f}_n e^{i\vec{k}_{\mathrm{Bl}}^{(n)}\vec{r}}. \end{aligned} \quad (1)$$

The latter equality in Eq. (1) presents this eigensolution as an infinite sum of plane waves with wave vectors differing from each other by reciprocal-lattice vectors $\vec{G}_n$. $\vec{k}_{\mathrm{Bl}}$ is the Bloch wave vector. Inside a photonic crystal all these plane waves are not independent. The relation between their amplitudes $\vec{f}_n$ is determined by mutual rescattering of the waves.

In the simplest case that of a one-dimensional photonic crystal made up of isotropic materials, the electric field of the wave traveling perpendicular to the layers, say along the $z$ axis ($x$ and $y$ axes chosen parallel to the layers as in Fig. 1), is governed by the following equation:

$$\frac{d^2\vec{E}}{dz^2} + k_0^2\varepsilon(z)\vec{E} = 0. \quad (2)$$

Here $k_0=(1/c)\omega$ is a reduced frequency or free space wave number. By substituting Eq. (1) into Eq. (2) one may get an infinite system of linear equations,

 

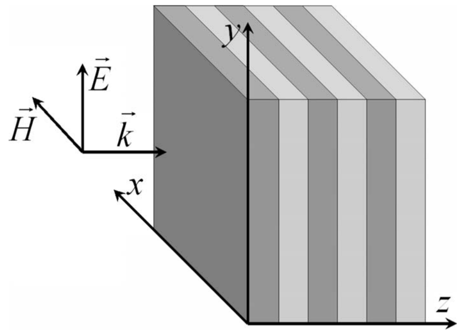


FIG. 1. Structure under consideration: layered structure and normal incidence case.

$$(\vec{k}_{\rm Bl}+\vec{G}_n)^2\vec{f}_n - k_0^2\sum_{n'}\varepsilon_{n-n'}\vec{f}_{n'} = 0, \qquad (3)$$

where $\varepsilon(z)=\Sigma_n\varepsilon_n e^{i\vec{G}_n z}$ and $\vec{G}_n=n\vec{G}$ are the reciprocal-lattice vectors.

We can recast Eq. (3) as

$$\vec{f}_n = \frac{k_0^2\sum\limits_{n'\neq n}\varepsilon_{n-n'}\vec{f}_{n'}}{(\vec{k}_{\rm Bl}+n\vec{G})^2 - k_0^2\varepsilon_0}.$$

Under the simultaneous fulfillment of conditions $(\vec{k}_{\rm Bl}-\vec{G})^2-k_0^2\varepsilon_0\approx 0$ and $k_{\rm Bl}^2-k_0^2\varepsilon_0\approx 0$ the denominator for $n=0$ and $n=-1$ simultaneously tends to zero. In this case, there are only two dominant plane harmonics with amplitudes $\vec{f}_0$ and $\vec{f}_{-1}$ and wave vectors $k_{\rm Bl}^{(0)}=k_{\rm Bl}$ and $k_{\rm Bl}^{(-1)}=(k_{\rm Bl}-G)\approx -k_{\rm Bl}$.[12] Thus, to first order in perturbation theory (with respect to $\varepsilon_1$) one can confine oneself to consideration of only these harmonics[10] and arrive at the following simple system of two linear equations:[13]

$$(k_{\rm Bl}^2-k_0^2\varepsilon_0)\vec{f}_0 - k_0^2\varepsilon_1\vec{f}_{-1} = 0,$$

$$[(k_{\rm Bl}-G)^2-k_0^2\varepsilon_0]\vec{f}_{-1} - k_0^2\varepsilon_1^*\vec{f}_0 = 0. \qquad (4)$$

For the case of a homogeneous medium ($\varepsilon_1=0$) we get two independent equations, the eigensolution of which yield the wave numbers of independent plane waves with amplitudes $\vec{f}_0$ and $\vec{f}_{-1}$. A nonzero $\varepsilon_1$ links both equations and relates $\vec{f}_0$ and $\vec{f}_{-1}$.

A nontrivial solution to Eq. (4) exists if and only if

$$\det\begin{vmatrix} k_{\rm Bl}^2-k_0^2\varepsilon_0 & -k_0^2\varepsilon_1 \\ -k_0^2\varepsilon_{-1} & (k_{\rm Bl}-G)^2-k_0^2\varepsilon_0 \end{vmatrix} = (k_{\rm Bl}^2-k_0^2\varepsilon_0)[(k_{\rm Bl}-G)^2-k_0^2\varepsilon_0] - k_0^4|\varepsilon_1|^2 = 0. \qquad (5)$$

By substitution $k_{\rm Bl}-G/2\rightarrow x$ this fourth-order equation may be reduced to a biquadratic equation that can be easily solved: $x^2=k_0^2\varepsilon_0+G^2/4\pm\sqrt{k_0^2\varepsilon_0G^2+k_0^4|\varepsilon_1|^2}$ but only $x^2=k_0^2\varepsilon_0+G^2/4-\sqrt{k_0^2\varepsilon_0G^2+k_0^4|\varepsilon_1|^2}\approx(k_0\sqrt{\varepsilon_0}-G/2)^2-\frac{k_0^2|\varepsilon_1|^2}{2\sqrt{\varepsilon_0}G}$ may simultaneously correspond to conditions $(\vec{k}_{\rm Bl}-\vec{G})^2-k_0^2\varepsilon_0\approx 0$ and $k_{\rm Bl}^2-k_0^2\varepsilon_0\approx 0$.

This solution predicts the existence of frequency bands around $k_0^{(0)}\sqrt{\varepsilon_0}=G/2$ where $x^2<0$ and $k_{\rm Bl}$ is complex. Thus this characteristic equation predicts the existence of a band gap.

The band-gap bandwidth is[10]

$$\Delta k_0 = \frac{|\varepsilon_1|k_0^{(0)}}{n_0^2},$$

where $n_0=\sqrt{\varepsilon_0}$ and $k_0^{(0)}=\frac{G}{2n_0}$ is the reduced frequency that corresponds to the condition $k_{\rm Bl}-G=-k_{\rm Bl}$ at $\varepsilon_1=0$ and is the band-gap center frequency.

So, we see that a periodic perturbation of the diagonal elements of the permittivity tensor induces a strong Bragg resonance among the plane waves, activating even those that cannot propagate in an unperturbed medium. At the boundary of the Brillouin zone ($2k_{\rm Bl}=G$) the Bloch wave mainly consists of two harmonics, which in a previously homogeneous medium traveled freely in opposite directions as plane waves. Since the amplitudes of these harmonics are identical they transfer identical energy in opposite directions. Thus, the Bragg resonance results in the absence of a general energy transfer throughout the system and hence in the formation of optical band gaps.

In a one-dimensional photonic crystal made up of isotropic components, the waves of any polarization, which travel in a normal direction to the layers acquire the same band structure. There is polarization degeneracy. If one of the components is a uniaxial crystal with anisotropy axis parallel to the layers, the degeneracy is removed and the ordinary and extraordinary Bloch waves induce different band gaps that may partially intersect. The magnetization of a photonic crystal made up of isotropic and magneto-optic materials results in a similar pattern: the Bloch waves of different polarization have different band gaps slightly shifted in frequency.

In all these cases the band gaps are formed at the Brillouin-zone boundary. From a mathematical point of view this follows from the fact that after choosing a proper basis (linearly polarized waves for the case of an anisotropic photonic crystal and right- and left-circularly polarized waves for the case of magnetophotonic crystal) the systems are described by Eq. (2) with a proper scalar periodic function $\varepsilon(z)$. This fact is a consequence of the conservation of symmetry throughout the system.

## III. BAND GAPS IN AN ANISOTROPIC MAGNETOPHOTONIC CRYSTAL

Photonic crystals made up of anisotropic materials having different anisotropy axes directions in different layers[14–18] or consisting of anisotropic and magneto-optic materials[19–24] may exhibit band gaps of a new type. To understand the formation of such band gaps let us consider a one-dimensional photonic crystal made up of anisotropic magneto-optic layers.[20,21] In this case Eq. (3) looks as follows: $(\vec{k}_{\rm Bl}+\vec{G}_n)^2\vec{f}_n-k_0^2\Sigma_{n'}\hat{\varepsilon}_{n-n'}\vec{f}_{n'}=0$ but contrary to Eq. (3) the tensor $\hat{\varepsilon}_n$ has also off-diagonal components. Thus,

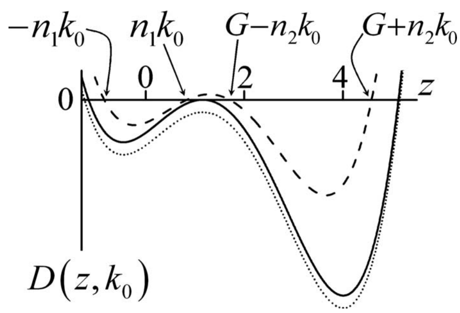


FIG. 2. The polynomial function $D(z,k_0)$ at different frequencies. The dashed line corresponds to an unperturbed case at some frequency $k_0 \neq k_0^{(0)}$, the solid line corresponds to an unperturbed case at frequency $k_0^{(0)}$, and the dotted line corresponds to a perturbed case at frequency $k_0^{(0)}$.

$$\hat{\varepsilon}(z) = \begin{pmatrix} \varepsilon_{xx}(z) & -ig(z) & 0 \\ ig(z) & \varepsilon_{yy}(z) & 0 \\ 0 & 0 & \varepsilon_{zz}(z) \end{pmatrix}, \tag{6}$$

where $\varepsilon_{aa}(z)=\Sigma_j \varepsilon_{aa,j} e^{ijGz}$, $a=x, y, z$, and $g(z)=\Sigma_j g_j e^{ijGz}$. This case differs from the case discussed in the previous section only in the form of the unperturbed dielectric tensor and can be realized by magnetizing a magneto-optic medium in the $z$ direction. In a Cartesian coordinate system (that is, with basis vectors $\hat{e}_x, \hat{e}_y$) Maxwell's equations reduce to

$$\begin{cases} \dfrac{d^2E_x}{dz^2} + k_0^2\varepsilon_{xx}(z)E_x - ik_0^2 g(z)E_y = 0 \\ \dfrac{d^2E_y}{dz^2} + k_0^2\varepsilon_{yy}(z)E_y + ik_0^2 g(z)E_x = 0. \end{cases} \tag{7}$$

In the unperturbed case we have propagation in a homogeneous anisotropic magneto-optic medium,

$$\begin{cases} \dfrac{d^2E_x}{dz^2} + k_0^2\varepsilon_{xx,0}E_x - ik_0^2 g_0 E_y = 0 \\ \dfrac{d^2E_y}{dz^2} + k_0^2\varepsilon_{yy,0}E_y + ik_0^2 g_0 E_x = 0. \end{cases} \tag{8}$$

Equation (8) has two independent eigensolutions traveling forward in the positive $z$ direction. These eigensolutions can be described in terms of their wave vectors and polarizations. We have right-elliptically polarized waves $\hat{e}_x + i\varphi\hat{e}_y$ with wave vector

$$\left(\frac{k_1}{k_0}\right)^2 = n_1^2 = \varepsilon + \sqrt{\delta^2 + g_0^2}, \tag{9a}$$

and left-elliptically polarized waves $i\varphi\hat{e}_x + \hat{e}_y$ with wave vector

$$\left(\frac{k_2}{k_0}\right)^2 = n_2^2 = \varepsilon - \sqrt{\delta^2 + g_0^2}, \tag{9b}$$

where $\varepsilon = \frac{\varepsilon_{xx,0}+\varepsilon_{yy,0}}{2}$, $\delta = \frac{\varepsilon_{xx,0}-\varepsilon_{yy,0}}{2}$, $\varphi = \frac{\sqrt{\delta^2+g_0^2}-\delta}{g_0}$, and $\hat{e}_x, \hat{e}_y$ are unit vectors in the $x$ and $y$ directions, respectively.

The basis set $\hat{e}_x, \hat{e}_y$ is not the most suitable for studying Eq. (7) because even in the unperturbed case the system of equations does not separate into two independent equations. Usually one chooses a basis set of elliptically polarized waves, where $E_r = E_x - i\varphi E_y$ and $E_l = E_y - i\varphi E_x$, in order to simplify Eq. (8). Considering Eq. (7) in this basis, we get

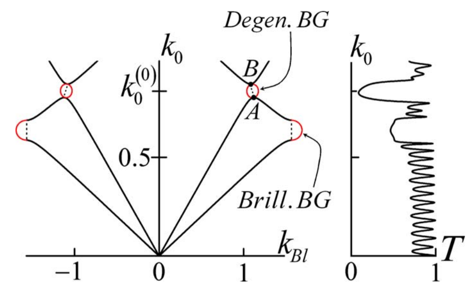


FIG. 3. (Color online) Dispersion curve in the reduced Brillouin-zone scheme and corresponding transmission coefficient $T$ (right side of the graph); the vertical axis is the same for both sides of the graph. The straight dashed black line shows the real part of $k_{\mathrm{Bl}}$, while the curved line bridging the band gaps, displayed in red online, shows the imaginary part of $k_{\mathrm{Bl}}$. Within the band gaps the real part of $k_{\mathrm{Bl}}$ is shown by a dotted line. Frequency $k_0$ and Bloch wave vector $k_{\mathrm{Bl}}$ are measured in $d^{-1}$ units. To make the effect more prominent we use photonic crystal parameters for two layers of the same thickness $d$ with $\varepsilon_{xx}=2.0$ and $\varepsilon_{yy}=7.7$ for the first layer and $\varepsilon_{xx}=1.5$, $\varepsilon_{yy}=4.5$, and $g=0.7$ for the second. The total length of the photonic crystal is 20 layers.

$$\begin{cases} \dfrac{d^2E_r}{dz^2} + k_1^2E_r + k_0^2A_r(z)E_r + k_0^2 iB(z)E_l = 0 \\ \dfrac{d^2E_l}{dz^2} + k_2^2E_l + k_0^2A_l(z)E_l - k_0^2 iB(z)E_r = 0. \end{cases} \tag{10}$$

Here

$$A_r(z) = \frac{\Delta\varepsilon_{xx} + 2\Delta g\varphi + \varphi^2\Delta\varepsilon_{yy}}{1+\varphi^2},$$

$$A_l(z) = \frac{\Delta\varepsilon_{yy} + 2\Delta g\varphi + \varphi^2\Delta\varepsilon_{xx}}{1+\varphi^2},$$

$$B(z) = \frac{(\Delta\varepsilon_{xx} - \Delta\varepsilon_{yy})\varphi + \Delta g(\varphi^2-1)}{1+\varphi^2},$$

$$\Delta\varepsilon_{jj} = \varepsilon_{jj}(z) - \varepsilon_{jj,0},$$

and

$$\Delta g = g(z) - g_0,$$

with $k_1$ and $k_2$ as defined in Eq. (9). Functions $A_r, A_l, B$ are periodic with the periodicity of the lattice and can be considered as a perturbation of the system [the quantities are small due to the smallness of $\varepsilon_{xx}(z)-\varepsilon_{xx,0}$, $\varepsilon_{yy}(z)-\varepsilon_{yy,0}$, and $g(z)-g_0$].

The solution to Eq. (10) is a Bloch wave $\vec{E} = \binom{E_r}{E_l} = \Sigma_j \binom{R_j}{L_j} e^{i(k_{\mathrm{Bl}}+jG)z}$. If we take into account the periodicity of $A_r, A_l, B$, we get

$$A_r(z)=\sum_j A_{r,j}e^{ijGz},\quad A_l(z)=\sum_j A_{l,j}e^{ijGz},$$

and

$$B(z)=\sum_j B_j e^{ijGz},$$

where

$$A_{r,j}=\frac{\varepsilon_{xx,j}+2\varphi g_j+\varphi^2\varepsilon_{yy,j}}{1+\varphi^2},$$

$$A_{l,j}=\frac{\varphi^2\varepsilon_{xx,j}+2\varphi g_j+\varepsilon_{yy,j}}{1+\varphi^2},$$

$$B_j=\frac{(\varepsilon_{xx,j}-\varepsilon_{yy,j})\varphi+g_j(\varphi^2-1)}{1+\varphi^2},$$

and

$$A_{r,0}=A_{l,0}=B_0=0.$$

While $A_l(z)$, $A_r(z)$, and $B(z)$ are real, then $A_{r,j}=A^*_{r,-j}$, $A_{l,j}=A^*_{l,-j}$, and $B_j=B^*_{-j}$.

Equation (10) reduces to

$$\begin{cases}-(k_{\mathrm{Bl}}+jG)^2R_j+k_1^2R_j+k_0^2\sum_{j'}A_{r,j-j'}R_{j'}+k_0^2 i\sum_{j'}B_{j-j'}L_{j'}=0\\ -(k_{\mathrm{Bl}}+jG)^2L_j+k_2^2L_j+k_0^2\sum_{j'}A_{l,j-j'}L_{j'}-k_0^2 i\sum_{j'}B_{j-j'}R_{j'}=0.\end{cases}\qquad(11)$$

We can recast Eq. (11) as

$$\begin{cases}R_j=\dfrac{k_0^2\sum_{j'}A_{r,j-j'}R_{j'}+k_0^2 i\sum_{j'}B_{j-j'}L_{j'}}{(k_{\mathrm{Bl}}+jG)^2-k_1^2}\\ L_j=\dfrac{k_0^2\sum_{j'}A_{l,j-j'}L_{j'}-k_0^2 i\sum_{j'}B_{j-j'}R_{j'}}{(k_{\mathrm{Bl}}+jG)^2-k_2^2}.\end{cases}$$

Under the simultaneous fulfillment of conditions $(\vec{k}_{\mathrm{Bl}}-\vec{G})^2-k_1^2\approx 0$ and $k_{\mathrm{Bl}}^2-k_1^2\approx 0$ the denominator for $j=-1$ and $j=0$ simultaneously tends to zero for $R_{-1}$ and $R_0$. Analogously, for $L_{-1}$ and $L_0$ under the simultaneous fulfillment of conditions $(\vec{k}_{\mathrm{Bl}}-\vec{G})^2-k_2^2\approx 0$ and $k_{\mathrm{Bl}}^2-k_2^2\approx 0$. These cases are identical to the one considered in the previous section.

However if $k_1\neq k_2$ one may find additional resonance conditions $(\vec{k}_{\mathrm{Bl}}-\vec{G})^2-k_1^2\approx 0$ and $k_{\mathrm{Bl}}^2-k_2^2\approx 0$ [or $k_{\mathrm{Bl}}^2-k_1^2\approx 0$ and $(\vec{k}_{\mathrm{Bl}}-\vec{G})^2-k_2^2\approx 0$] which result in the simultaneous growth of $R_{-1}$ and $L_0$ (correspondingly $R_0$ and $L_{-1}$).

Thus to consider both types of resonance conditions we restrict ourselves to consideration of four dominant harmonics: $R_0$, $R_{-1}$, $L_0$, and $L_{-1}$. Neglecting other small harmonics Eq. (11) turns into Eq. (12) below, which is, in general, analogous to Eq. (4)

$$\begin{cases}(k_1^2-k_{\mathrm{Bl}}^2)R_0+k_0^2A_{r,1}R_{-1}+k_0^2 iB_1L_{-1}=0\\ [k_1^2-(k_{\mathrm{Bl}}-G)^2]R_{-1}+k_0^2A_{r,-1}R_0+k_0^2 iB_{-1}L_0=0\\ (k_2^2-k_{\mathrm{Bl}}^2)L_0+k_0^2A_{l,1}L_{-1}-k_0^2 iB_1R_{-1}=0\\ [k_2^2-(k_{\mathrm{Bl}}-G)^2]L_{-1}+k_0^2A_{l,-1}L_0-k_0^2 iB_{-1}R_0=0.\end{cases}\qquad(12)$$

Here there are two possible Bragg resonance cases. The first case is a Bragg resonance when incident and reflected harmonics are of the same polarization, namely, between $R_0$ and $R_{-1}$ or between $L_0$ and $L_{-1}$. These Bragg resonances correspond to the formation of band gaps at the boundary of the Brillouin zones. In fact, the Bragg resonance between harmonics $R_0$ and $R_{-1}$ occurs if $k_1^2-k_{\mathrm{Bl}}^2\approx 0$ and $k_1^2-(k_{\mathrm{Bl}}-G)^2\approx 0$. In this case Eq. (12) becomes

$$\begin{cases}(k_1^2-k_{\mathrm{Bl}}^2)R_0+k_0^2A_{r,1}R_{-1}=0\\ [k_1^2-(k_{\mathrm{Bl}}-G)^2]R_{-1}+k_0^2A_{r,-1}R_0=0,\end{cases}$$

which is identical to Eq. (4).

The corresponding characteristic equation,

$$(k_{\mathrm{Bl}}^2-k_1^2)[(k_{\mathrm{Bl}}-G)^2-k_1^2]-k_0^4|A_{r,1}|^2=0,$$

is biquadratic, and therefore it may be easily solved. Its solution (in the same way as in the previous chapter) predicts the frequencies at which the Bloch wave vector is a complex quantity and thus predicts the formation of band gaps.

The band-gap bandwidth is

$$\Delta k_0=\frac{|A_{r,1}|k_0^{(0)}}{n_1^2},\qquad(13)$$

where $k_0^{(0)}=\frac{G}{2n_1}$ with $n_1$ as defined in Eq. (9). Thus one may conclude that the Bragg resonance between harmonics $R_0$ and $R_{-1}$ (or between $L_0$ and $L_{-1}$) results in band gaps at the edges of the Brillouin zone as in the previous chapter. Below we will refer to such band gaps as Brillouin BGs, where BG stands for band gap. The second case of resonant Bragg reflection occurs when incident and reflected harmonics are cross polarized, namely, $R_0$ and $L_{-1}$ and $L_0$ and $R_{-1}$. In this case, the Bragg reflection results in the formation of band gaps of new type. These band gaps form inside the Brillouin zone under fulfillment of the Bragg conditions: $k_1^2-k_{\mathrm{Bl}}^2\approx 0$,

$k_2^2-(k_{\rm Bl}-G)^2\approx 0$, $k_2^2-k_{\rm Bl}^2\approx 0$, and $k_1^2-(k_{\rm Bl}-G)^2\approx 0$, respectively.

The Bragg reflection of cross polarized harmonics $R_0$ and $L_{-1}$ is essential if $k_1^2-k_{\rm Bl}^2\approx 0$ and $k_2^2-(k_{\rm Bl}-G)^2\approx 0$. Confining ourselves to consideration of two of these harmonics we arrive at the following system of equations:

$$\begin{cases}(k_{\rm Bl}^2-k_1^2)R_0-ik_0^2B_1L_{-1}=0\\ [(k_{\rm Bl}-G)^2-k_2^2]L_{-1}+ik_0^2B_{-1}R_0=0.\end{cases}\tag{14}$$

Here $k_1=n_1k_0$ and $k_2=n_2k_0$ are the wave vectors corresponding to unperturbed (homogeneous anisotropic and gyrotropic) media.

This system of equations is also similar to Eq. (4) with the only difference that $k_1$ and $k_2$ are no longer equal to $k_0\sqrt{\varepsilon_0}$ but possess different values. This difference plays a dramatic role in the solution of the characteristic equation:[25]

$$\det\begin{vmatrix}k_2^2-(k_{\rm Bl}-G)^2 & -ik_0^2B_1\\ ik_0^2B_{-1} & k_1^2-k_{\rm Bl}^2\end{vmatrix}$$
$$=[k_2^2-(k_{\rm Bl}-G)^2](k_1^2-k_{\rm Bl}^2)-k_0^4|B_1|^2=0.\tag{15}$$

Because $k_1\neq k_2$ this characteristic equation, unlike Eq. (5), cannot be reduced to a simple biquadratic equation with respect to $k_{\rm Bl}$ thus demanding a more complicated analysis. To make such an analysis we pay attention to the properties of the polynomial function $D(z,k_0)=[n_2^2k_0^2-(z-G)^2](n_1^2k_0^2-z^2)-k_0^4|B_1|^2$ for different frequencies. The roots of $D(z,k_0)=0$ determine the dispersion relation.

In the unperturbed case ($B_1=0$) for arbitrary frequency $k_0$, Eq. (15) has four solutions: $k_{\rm Bl1,3}=\pm k_1=\pm n_1k_0$ and $k_{\rm Bl2,4}=G\pm k_2=G\pm n_2k_0$ which correspond to intersections of the graph of $D(z,k_0)$ with the abscissa axis (see Fig. 2 dashed line). But we have to take into account the solutions that satisfy conditions $k_1^2-k_{\rm Bl}^2\approx 0$ and $k_2^2-(k_{\rm Bl}-G)^2\approx 0$ under which Eq. (15) is derived. The simultaneous fulfillment of these conditions happens at a particular frequency $k_0^{(0)}=\frac{G}{n_1+n_2}$ and for two Bloch vectors $k_{\rm Bl1},k_{\rm Bl2}$ only. At this particular frequency $k_{\rm Bl1}=k_{\rm Bl2}$. Thus, the polynomial function $D(k_{\rm Bl})$ at the frequency $k_0^{(0)}$ has double roots (see Fig. 2 solid line). Adding any small negative term $-k_0^4|B_1|^2$ even if infinitesimally small (see Fig. 2 dotted line) results in shifting down the whole graph and, as a consequence, in the elimination of the crossing of $D(z,k_0)$ with the horizontal axis. This means that the roots acquire complex values. We can conclude that an anisotropic perturbation in the gyrotropic medium causes the formation of the new band gap.

Returning now to the other condition for a Bragg resonance, between of $R_{-1}$ and $L_0$, one obtains the following equation:

$$\begin{cases}[(k_{\rm Bl}-G)^2-k_1^2]R_{-1}-ik_0^2B_{-1}L_0=0\\ (k_{\rm Bl}^2-k_2^2)L_0+ik_0^2B_1R_{-1}=0.\end{cases}$$

And its respective characteristic equation

$$\det\begin{vmatrix}k_1^2-(k_{\rm Bl}-G)^2 & -ik_0^2B_1\\ ik_0^2B_1 & k_2^2-k_{\rm Bl}^2\end{vmatrix}$$
$$=[k_1^2-(k_{\rm Bl}-G)^2](k_2^2-k_{\rm Bl}^2)-k_0^4|B_1|^2=0,\tag{16}$$

which is similar to Eq. (15). The only difference is the exchange of the indices 1 and 2 in $k_i$. Thus, at the same frequency $k_0^{(0)}=\frac{G}{n_1+n_2}$ a new band gap forms. One may find that by the substitution $k_{\rm Bl}\to G-k_{\rm Bl}$ this equation turns into Eq. (15). Therefore since the solution of Eq. (15) has a nonzero imaginary part then the solution of Eq. (16) also has a nonzero imaginary part and these two equations predict the same band gaps. Particularly these equations predict the same band-gap bandwidth.

In general the solution of Eq. (15) is quite complicated but in the special case where $|B_1|\ll|n_1-n_2|$ it is possible to find the following approximate band-gap bandwidth:

$$\Delta k_0=\frac{|B_1|k_0^{(0)}}{n_1n_2},\tag{17}$$

where $k_0^{(0)}=\frac{G}{n_1+n_2}$ with $n_1$ and $n_2$ as defined in Eq. (9).

A comparison of Eqs. (13) and (17) shows that while the first type of Bragg resonance is caused by terms $A_i$, the second type of Bragg resonance is caused by the term $B_i$. Now one may see the convenience of using the formulation presented in Eq. (12)—we have separated the different types of resonance. Different resonances are determined by the different terms. Particularly for $B=0$,

$$\frac{\Delta\varepsilon_{xx}(z)-\Delta\varepsilon_{yy}(z)}{\Delta\varepsilon_{xx,0}-\Delta\varepsilon_{yy,0}}=\frac{\Delta g(z)}{g_0}.$$

This condition means that if $[\varepsilon_{xx}(z)-\varepsilon_{yy}(z)]/g(z)=\text{const}$ (Ref. 21) then $B=0$ and there is no band gap of a new type.

It should be pointed out that these new band gaps (defined in terms of $B$) possess an unusual property, namely, polarization degeneracy. Indeed Bragg resonances between right-polarized harmonics $R_0$ and $R_{-1}$ become essential at frequency $k_0^{(0)}=\frac{G}{2n_1}$, while Bragg resonances between left-polarized harmonics $L_0$ and $L_{-1}$ become essential at frequency $k_0^{(0)}=\frac{G}{2n_2}$. In general, when $n_1\neq n_2$, these frequencies are different and their band-gap bandwidths are also different. At the same time Bragg resonances between $R_0$ and $L_{-1}$ and between $L_0$ and $R_{-1}$ always occur simultaneously at the same frequency $k_0^{(0)}=\frac{G}{n_1+n_2}$. Moreover their band-gap bandwidths are also the same.

We may conclude that perturbation theory predicts the formation of band gaps of the same width around the same frequency $k_0^{(0)}=\frac{G}{n_1+n_2}$ for each polarization. So this band gap appears simultaneously for both polarization states, or in other words this formation is degenerate with respect to polarization and we will refer to such band gaps as degenerate BGs.

## IV. PROPERTIES OF DEGENERATE BAND GAPS

We have seen that perturbation theory predicts the formation of degenerate band gaps around frequency $k_0^{(0)}$. These band gaps arise under the fulfillment of general Bragg conditions between harmonics of different polarization $k_0^{(0)}n_1 + k_0^{(0)}n_2 = G$. To study the properties of degenerate band gaps we have calculated the exact band structure for the previously considered one-dimensional photonic crystals. To calculate the dispersion curves and band structure for such magnetophotonic crystals we now employ the transfer-matrix method.[15,20,26,27]

At optical frequencies for present-day materials the bandwidth of the gyrotropic degenerate band gap is small due to the smallness of the off-diagonal element $g$ in the dielectric tensor. For example, for a photonic crystal with bilayer unit cell consisting of bismuth-substituted yttrium iron garnet (typical dielectric tensor components $\varepsilon_{xx}=5.6$, $\varepsilon_{yy}=5.6$, and $g=0.018$), with layer thickness $d$, and $CaCO_3$ ($\varepsilon_{xx}=2.2, \varepsilon_{yy}=2.8$), with layer thickness $2d$, the gyrotropic degenerate band-gap bandwidth at saturation magnetization is only $\Delta k_0 = 0.8\times 10^{-4} d^{-1}$ around frequency $k_0=1.140 d^{-1}$ with the imaginary part of the Bloch wavevector equal to $\mathrm{Im}(k_{\mathrm{Bl}}) = 0.2\times 10^{-3} d^{-1}$ at the center of the band gap. In spite of the smallness of $\Delta k_0$, this band gap is degenerate with respect to polarization (just as predicted by perturbation theory).

It is possible, however, to significantly enhance the net Faraday rotation delivered by the photonic crystal through the use of photon-trapping structures.[4,6,28] These structures yield a larger effective gyrotropy parameter $g$ thus increasing the bandwidth of the gyrotropic degenerate band gap.[29] To simplify our considerations we will not concern ourselves with these structures and will focus our attention on the fundamental properties of the degenerate band gap.

Below, in order to highlight the properties of the gyrotropic degenerate band gap and for ease of comparison with those of the Brillouin band gap, the anisotropy contrast and gyrotropy parameters have been chosen artificially large. Figure 3 plots the dispersion curve for a photonic crystal with unit-cell structure consisting of two layers with parameters $\varepsilon_{xx}=2.0$ and $\varepsilon_{yy}=7.7$ for the first layer and $\varepsilon_{xx}=1.5$, $\varepsilon_{yy}=4.5$, and $g=0.7$ for the second. Both layers have the same thickness and the total length of the photonic crystal is 20 layers.

First of all let us consider the character of the dispersion curves in a reduced Brillouin-zone scheme (Fig. 3). In Fig. 3 one can observe besides the Brillouin band gaps at the edges of the Brillouin zone (referred to as Brillouin BG) the presence of band gaps around frequency $k_0^{(0)}$ [corresponding to the fulfillment of conditions $k_1^2 - k_{\mathrm{Bl}}^2 \approx 0$ and $k_2^2 - (k_{\mathrm{Bl}} - G)^2 \approx 0$]. These band gaps coincide exactly in frequency and wave-vector coordinates for both Bloch wave polarization states, in accordance with the predictions of perturbation theory on the formation of degenerate band gaps with respect to polarization.

The degenerate band gap in Fig. 3 forms inside the Brillouin zone, contrary to the Brillouin BG which forms at the boundary of the Brillouin zone. To clarify this point one may consider dispersion curves (see Fig. 4) for the same photonic crystal structure whose dispersion was depicted in Fig. 3 but with the gyrotropy turned off, namely, $g=0$ (respectively, term $B_i=0$ and there are no degenerate band gaps). This condition can be achieved by switching off the magnetic field in the absence of hysteresis. In that case the frequency $k_0^{(0)}$ corresponds to the frequency of effective isotropy,[19] namely, the intersection point of dispersion curves. Thus, in the presence of gyrotropy a degenerate band gap forms inside the Brillouin zone around a frequency of effective isotropy.

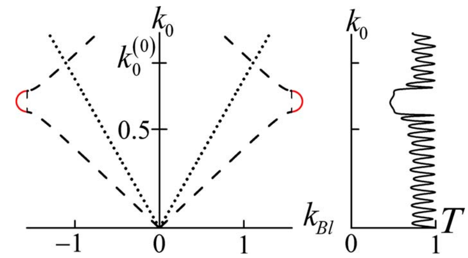


FIG. 4. (Color online) Dispersion curve in the reduced Brillouin-zone scheme and corresponding transmission coefficient $T$ (right side of the graph) for a photonic crystal structure with unit cell consisting of two layers of the same thickness $d$ with $\varepsilon_{xx}=2.0$ and $\varepsilon_{yy}=7.7$ for the first layer and $\varepsilon_{xx}=1.5$, $\varepsilon_{yy}=4.5$, and $g=0$ for the second. The total length of the photonic crystal is 20 layers. The figure shows that in the absence of gyrotropy, a condition that may be induced by switching off a magnetizing field on a suitably prepared magneto-optic medium, the degenerate band gap is suppressed, while the Brillouin band gap persists. Notice that the dispersion curves for different Bloch waves intersect at $k_0^{(0)}$.

If we return to Fig. 3 one can find that for each frequency $k_0$ there are four Bloch wave-vector solutions inside the first Brillouin zone. Within the degenerate BG these wave vectors take the special form $a(k_0)\pm ib(k_0)$ and $-a(k_0)\pm ib(k_0)$, where $a(k_0)$ and $b(k_0)$ are real positive quantities. In particular two of these Bloch wave vectors, $k_{\mathrm{Bl}}=a+ib$ and $k_{\mathrm{Bl}}=-a+ib$ correspond to two different evanescent Bloch waves, while Bloch wave vectors $k_{\mathrm{Bl}}=a-ib$ and $k_{\mathrm{Bl}}=-a-ib$ correspond to evanescent Bloch waves in the opposite $z$-axis direction.

As was pointed out above, the main peculiarity of such band gaps (and their difference with Brillouin band gaps) is the polarization degeneracy at Bragg reflection. Points A and B in Fig. 3 show the edges of the degenerate BG $[b(k_0)=0]$. For each frequency coordinate at the band edges the corresponding Bloch waves have exactly the same Bloch wave vectors. This can be pictured by approaching the band edges from the passband side. For frequencies just below point A, or just above point B, there are two Bloch mode solutions with different Bloch wave vectors for each given frequency $k_0$. When the extremal points A and B are reached, these two solutions, corresponding to different Bloch waves with different polarization states, become degenerate in Bloch wave vector. At each frequency inside the band gap the real part of the wave vector $a(k_0)$ [shown as a black dashed bridge between points A and B (Ref. 30)] is the same for both polarizations. However, the imaginary parts of these wave vectors $\pm b(k_0)$ are different (Fig. 3 curved lines bridging the band gaps, shown in red online).[31]

Also it should be pointed out that bridge AB is not a vertical line. For frequencies inside the Brillouin BG the real part of the Bloch wave vector (vertical dashed line in Fig. 3) equals $G/2$ and is independent of the frequency, whereas for the degenerate BG the real part of the Bloch wave vector (dashed bridge AB) is a function of frequency.

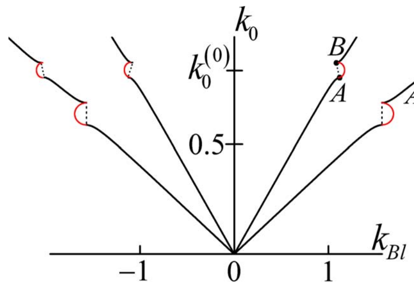


FIG. 5. (Color online) Dispersion curve in the enhanced Brillouin-zone scheme, the red line shows the imaginary part of $k_{\mathrm{Bl}}$. The parameters of the photonic crystal are for a unit cell consisting of two layers of the same thickness $d$ with $\varepsilon_{xx}=2.0$ and $\varepsilon_{yy}=7.7$ for the first layer and $\varepsilon_{xx}=1.5$, $\varepsilon_{yy}=4.5$, and $g=0.7$ for the second. The total length of the photonic crystal is 20 layers. These parameters are the same as for Fig. 3.

Another important feature of the degenerate band gap is its response to arbitrarily polarized light. Since the Brillouin BG for one polarization corresponds to a passing band for another, the transmittance for unpolarized light cannot be significantly less than 0.5 as shown in the transmittance plot of Fig. 3. On the other hand, the gyrotropic degenerate BG reflects back all polarizations simultaneously. Hence its light-blocking efficiency depends only on the reflectivity of the grating and can be made arbitrarily large by making the photonic crystal sufficiently long. Figure 3 shows the transmittance for unpolarized light for both band gaps.

To study the direction of propagation or evanescing of the Bloch waves it is useful to consider the dispersion curves in an enhanced Brillouin-zone scheme (Fig. 5), which also helps us to separate the dispersion curves according to polarization. In an enhanced Brillouin-zone scheme the Bloch wave vectors inside the degenerate band gap take the form $\pm[a(k_0)+ib(k_0)]$ and $\pm[G-a(k_0)+ib(k_0)]$. In such representation the wave vectors with positive value of the real part corresponds to evanescent waves.

It is easy to see that the degenerate BG results in curve discontinuities just inside the Brillouin zone. Furthermore since the bridge is positioned angularly then for one of the polarization states there is an interval in real wave vectors (bridge A′B′) with no allowed frequencies. At the same time for another polarization state there is a continuous interval of real wave vectors (bridge AB) with two frequencies corresponding to each wave vector. These properties are unusual for band gaps and cannot be realized for Brillouin band gaps.

## V. CONCLUSION

We observe that the magnetization of a one-dimensional photonic crystal made up of isotropic dielectric and/or isotropic magneto-optical materials results only in a shift of the dispersion curves for right- and left-circularly polarized waves in opposite directions. In other words one cannot attain a situation where at some frequency the dispersion curves for both right- and left-circularly polarized waves will shift from a passband into the band gap simultaneously. Thus, in a one-dimensional magnetophotonic crystal based on isotropic magneto-optical materials one cannot manipulate arbitrarily polarized light (or simultaneously both polarizations) by the use of an external magnetic field. This remark shows that the absence of polarization degeneracy restricts the range of applications of magnetophotonic crystals as tunable devices.

In the present paper we have shown that the combination of the following three properties: anisotropy, gyrotropy, and periodicity—can result in the formation of gyrotropic degenerate band gaps. These band gaps appear inside the Brillouin zone as a result of the Bragg resonance between local normal modes having different polarization states.

We have shown that the term degenerate (referred to the band gap) reveals itself in different properties: (1) the Bloch waves at the edges of the band gap are degenerate with respect to polarization, (2) inside the degenerate band gap the real part of the Bloch wave vector is the same for both solutions, and (3) the degenerate band gap appears simultaneously for both polarizations. At the same time, all these three properties are closely connected to each other and only occur in conjunction with each other.

Thereupon the degenerate band gap is of great interest. Indeed, on the one hand the degenerate band gap always appears simultaneously for both polarizations. On the other hand one may control its appearance in magnetophotonic crystals by the use of an external magnetic field [since its bandwidth is proportional to magnetization, see Eq. (17) and Refs. 19–24].

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[29] Particularly, one may consider a more complex structure for the photonic crystal with the unit cell in the form of a supercell consisting of an anisotropic layer adjacent to a quarter-wave-phase-shifted structure (a subcell) with the resonant cavity made up of bismuth-substituted yttrium iron garnet.
[30] This bridge AB starts at the upper point (A) of the lower branch of the dispersion curve and comes to an end at the lower point (B) of the upper branch of the dispersion curve.
[31] Moreover, it can be shown that we may have polarization degeneracy in Bloch wave vector in some continuous interval of frequencies if and only if we have polarization degeneracy at all frequencies. So, Figs. 4 and 5 from [P. Yeh, J. Opt. Soc. Am. **69**, 742 (1979)] which show only one imaginary root are not complete—inside the band gap there is no polarization degeneracy in the imaginary components of the wave vector.